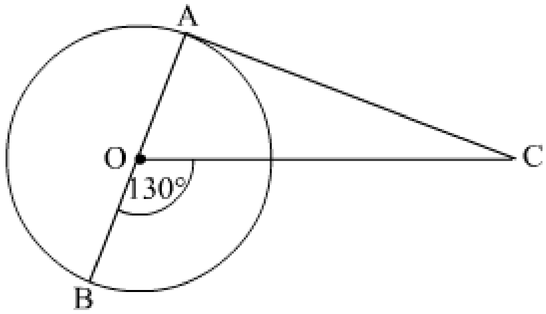
<u>**Section A**</u>

Q1 In Fig. 1, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130°$ v, the find $\angle ACO$.



Ans:- In the given figure, $\angle BOC = 130°$.

$\angle BOC$ and $\angle AOC$ form a linear pair.

So,

$\angle BOC + \angle AOC = 180°$

$\Rightarrow \angle AOC = 180° - 130° = 50°$

$Also, \angle CAO = 90°$. (Because the tangent at any point of a circle is perpendicular to the radius through the point of contact)

$In\ \Delta\ AOC,$

$\angle AOC + \angle CAO + \angle ACO = 180°$ (Angle sum property of a triangle)

$\Rightarrow 50° + 90° + \angle ACO = 180°$

$\Rightarrow \angle ACO = 40°$

$\therefore \angle ACO = 40°$

Q2 An observer, 1.7 m tall, is $20\sqrt{3}\ m$ away from a tower. The angle of elevation from the of observer to the top of tower is 30°. Find the height of tower.

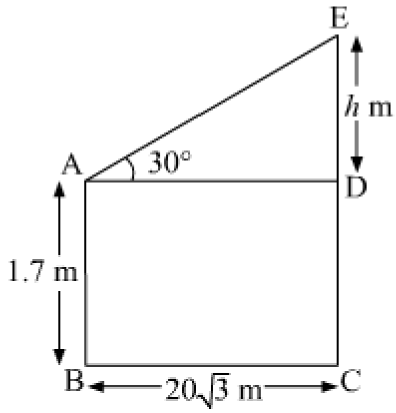
Ans: Let AB be the height of the observer and EC be the height of the tower.

Given:

AB = 1.7 m ⇒ CD = 1.7 m

$BC = 20\sqrt{3}\ m$

Let ED be *h* m.



In ?ADE,

$$tan 30° = \frac{ED}{AD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20\sqrt{3}}$$

$$\Rightarrow h = 20m$$

∴ EC = ED + DC = (*h* + 1.7) m = 21.7 m

Hence, the height of the tower is 21.7 m.

Q3 For what value of k will the consecutive terms 2k + 1, 3k + 3 and 5k − 1 form an A.P.?

Ans:- The given terms are 2*k* + 1, 3*k* + 3 and 5*k* − 1.

The differences between the consecutive terms are

$3k + 3 - (2k + 1) = k + 2 = d_1$

and

$5k - 1 - (3k + 3) = 2k - 4 = d_2$

If the given terms are in an AP, then

$d_1 = d_2$

$\Rightarrow k + 2 = 2k - 4$

$\Rightarrow k = 6$

Hence, the value of *k* for which the given terms are in an AP is 6.

Q4 20 tickets, on which numbers 1 to 20 are written, are mixed thoroughly and then a ticket is drawn at random out of them. Find the probability that the number on the drawn ticket is a multiple of 3 or 7.

Ans:- Total number of outcomes = 20

Possible outcomes = 3, 6, 7, 9, 12, 14, 15 and 18

∴ Number of favourable outcomes = 8

Probability = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$

∴ Probability that the number on the drawn ticket is a multiple of 3 or 7=820=25

Section B

Q5 A two-digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number.

Ans:- Let the digits of the required number be x and y.

Now, the required number is $10x + y$.

According to the question,

$10x + y = 4(x + y)$

So,

$6x - 3y = 0$

$\Rightarrow 2x - y = 0$

$x = \frac{y}{2}$ .....(1)

Also,

$10x + y = 3xy$ .....(2)

From (1) and (2), we get

$10\left(\frac{y}{2}\right) + y = 3\left(\frac{y}{2}\right)y$

$\Rightarrow 5y + y = \frac{3}{2}y^2$

$\Rightarrow 6y = \frac{3}{2}y^2$

$\Rightarrow y^2 - 4y = 0$

$\Rightarrow y(y-4) = 0$

$\Rightarrow y = 0, 4$

So, $x = 0$ for $y = 0$ and $x = 2$ for $y = 4$.
Hence, the required number is 24.

Q6 Find the ratio in which the point (−3, k) divides the line-segment joining the points (−5, −4) and (−2, 3). Also find the value of k.

Ans:- Suppose the point P(−3, *k*) divides the line segment joining the points A(−5, −4) and B(−2, 3) in the ratio *m* : 1.

Then, the coordinates of the point P will be $\left(\frac{-2m-5}{m+1}, \frac{3m-4}{m+1}\right)$

Also, it is given that the coordinates of the point P are (−3, *k*).

$\therefore \frac{-2m-5}{m+1} = -3 \text{ and } \frac{3m-4}{m+1} = k$

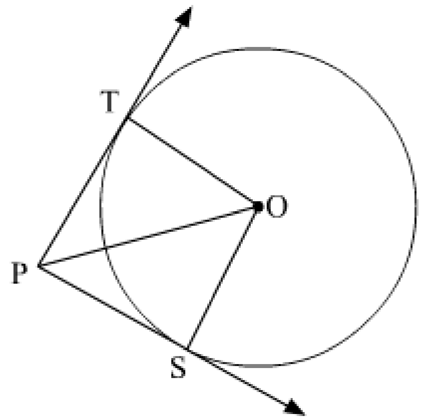
$\Rightarrow -2m-5 = -3 \text{ and } \frac{3m-4}{m+1} = k$

$\Rightarrow m = 2 \text{ and } k = \frac{3m-4}{m+1}$

$\Rightarrow m = 2 \text{ and } k = \frac{2}{3}$

Hence, the required ratio is 2 : 1 and the value of *k* is $\frac{2}{3}$.

Q7 In Fig. 2, from a point P, two tangents PT and PS are drawn to a circle with centre O such that ∠SPT = 120°, Prove that OP = 2PS.



Ans:- It is given that PS and PT are tangents to the circle with centre O. Also, ∠SPT = 120°.
To prove: OP = 2PS
Proof: In $\Delta PTO \text{ and } \Delta PSO,$
PT = PS (Tangents drawn from an external point to a circle are equal in length.)
TO = SO (Radii of the circle)
$\angle PTO = \angle PSO = 90^\circ$
(Tangent at any point of a circle is perpendicular to the radius through the point of contact.)
$\therefore \Delta PTO \cong \Delta PSO$ (By SAS congruency)
Thus,
$$\angle TPO = \angle SPO = \frac{120^\circ}{2} = 60^\circ$$
Now, in $\Delta PSO,$
$$\cos 60^\circ = \frac{PS}{PO}$$
$$\Rightarrow \frac{1}{2} = \frac{PS}{PO}$$
$$\Rightarrow PO = 2PS$$
Hence proved.

Q8 Prove that the points (2, −2), (−2, 1) and (5, 2) are the vertices of a right angled triangle. Also find the area of this triangle.
Ans:- Let A(2, −2), B(−2, 1) and C(5, 2) be the vertices of the given triangle.
Now,
$$AB = \sqrt{(1+2)^2 + (-2-2)^2} = \sqrt{25} = 5\, units$$

$$BC = \sqrt{(2-1)^2 + (5+2)^2} = \sqrt{50} = 5\sqrt{2}\ units$$

$$CA = \sqrt{(2+2)^2 + (5-2)^2} = \sqrt{25} = 5\ units$$

$$\therefore CA^2 + AB^2 = BC^2$$

Hence, by the converse of Pythagoras' theorem, $\angle A = 90°$.
ΔABC is right-angled.

Q9 If the ratio of sum of the first m and n terms of an AP is $m^2 : n^2$, show that the ratio of its m^{th} and n^{th} terms is (2m − 1) : (2n − 1).

Ans:- Let the first term and the common difference of the AP be *a* and *d*, respectively.
Therefore,
Sum of the first *m* terms of the AP,

$$S_m = \frac{m}{2}\left[2a + (m-1)d\right]$$

Sum of the first *n* terms of the AP, *Sn*=*n*2[2*a*+(*n*−1)*d*]
It is given that

$$\frac{S_m}{S_n} = \frac{\frac{m}{2}\left[2a+(m-1)d\right]}{n2\left[2a+(n-1)d\right]} = \frac{m^2}{n^2}$$

$$2a(n-m) = d(n-m)$$

$$\Rightarrow \Rightarrow 2a = d$$

Now,

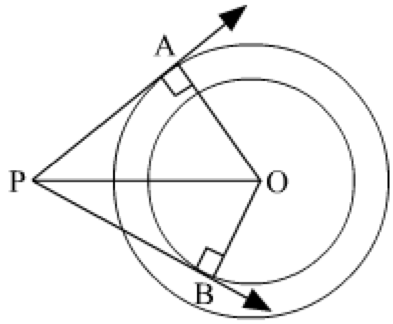
$$\frac{T_m}{T_n} = \frac{a+(m-1)d}{a+(n-1)d}$$

$$\Rightarrow \frac{T_m}{T_n} = \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a}$$

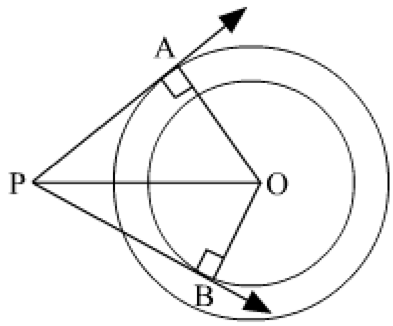
$$\Rightarrow \frac{Tm}{Tn} = \frac{2m-1}{2n-1}$$

Hence, the ratio of the m^{th} term to the n^{th} term is (2*m* − 1) : (2*n* − 1).

Q10 In fig. 3 are two concentric circles of radii 6 cm and 4 cm with centre O. If AP is a tangent to the larger circle and BP to the smaller circle and length of AP is 8 cm, find the length of BP.



Ans:-

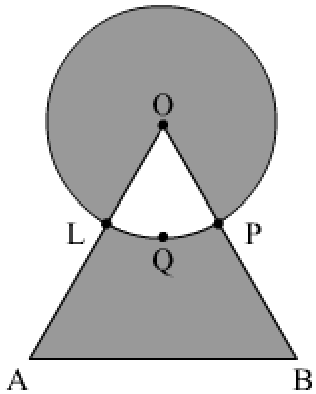


Given:
OA = 6 cm
OB = 4 cm
AP = 8 cm
Consider ΔOAP.
By Pythagoras' theorem, we have
$OA^2 + AP^2 = PO^2$
$\Rightarrow 6^2 + 8^2 = PO^2$
$\Rightarrow PO^2 = 100$
$\Rightarrow PO = 10\ cm$
Now, consider ?OBP.
By Pythagoras' theorem, we have
$OB^2 + BP^2 = PO^2$
$\Rightarrow 4^2 + BP^2 = 10^2$
$\Rightarrow BP^2 = 84$
$\Rightarrow BP = 2\sqrt{21} cm$
Hence, the length of BP is $2\sqrt{21}$ cm.

SECTION C

Q11 Find the area of shaded region in Fig. 4, where a circle of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12

cm. $\left(Use\ \pi = 3.14\ and\ \sqrt{3} = 1.73\right)$



Ans:- Given:
Radius of the circle (r) = 6 cm
Side of the equilateral triangle (a) = 12 cm
Now,
Area of the shaded region = Area of the circle with centre O + Area of the equilateral triangle OAB − 2(Area of the sector OLP)

$$= \pi r^2 + \frac{\sqrt{3}}{4}a^2 - 2\frac{\theta}{360°}\pi r^2$$

$$= \pi(6)^2 + \frac{\sqrt{3}}{4}(12)^2 - 2\frac{60°}{360°}\pi(6)^2$$

(Because OAB is an equilateral triangle)

$$= 36\pi + \frac{\sqrt{3}}{4}(144) - 2\frac{60°}{360°}\pi(36)$$

$$= 24\pi + 36\sqrt{3}$$

$$= 24 \times 3.14 + 36 \times 1.73$$

$$= 75.36 + 62.28 = 137.64\,cm^2$$

Q12 A hemispherical tank, of diameter 3 m, is full of water. It is being emptied by a pipe at the rate of 347 litre per second. How much time will it take to make the tank half empty? $\left[Use\ \pi = \frac{22}{7}\right]$

Ans:- Radius of the hemispherical tank = $\frac{3}{2}m$

Volume of the tank = $\frac{2}{3} \times \frac{22}{7} \times \left(\frac{3}{2}\right)^3 = \frac{99}{14}m^3$

So,

Amount of water to be taken out of the tank

$$= \frac{1}{2}\times\frac{99}{14}m^3 = \frac{99}{28}\times 1000\, litres = \frac{99000}{28}L$$

Since $\frac{25}{7}$ litres of water is taken out in 1 second,

$\frac{99000}{28}\, litres$ of water will be taken out in $\frac{99000}{28}\times\frac{7}{25}$ seconds, i.e. 16.5 minutes.

Q13 If the point C (–1, 2) divides internally the line-segment joining the points A (2, 5) and B (*x*, *y*) in the ratio 3 : `4, find the value of $x^2 + y^2$.

Ans:- It is given that the point C(–1, 2) divides the line segment joining the points A(2, 5) and B(*x*, *y*) in the ratio 3 : 4 internally.

Using the section formula, we get

$$(-1,2) = \left(\frac{3\times x + 4\times 2}{3+4}, \frac{3\times y + 4\times 5}{3+4}\right)$$

$$\Rightarrow (-1,2) = \left(\frac{3x+8}{7}, \frac{3y+20}{7}\right)$$

$$\Rightarrow 3x + 87 = -1 \; and \; \frac{3y+20}{7} = 2$$

$$\Rightarrow 3x + 8 = -7 \; and \; 3y + 20 = 14$$

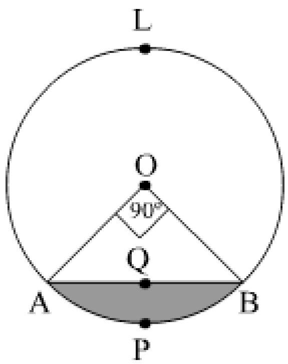
$$\Rightarrow 3x = -15 \; and \; 3y = -6$$

$$\Rightarrow x = -5 \; and y = -2$$

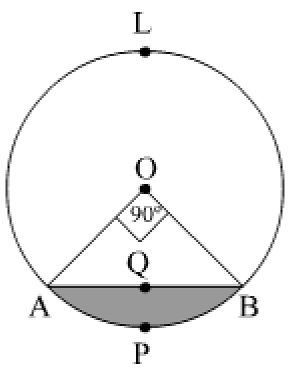
$$\therefore x^2 + y^2 = 25 + 4 = 29$$

Hence, the value of $x^2 + y^2$ is 29.

Q14 In fig. 5 is a chord AB of a circle, with centre O and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment AQBP. Hence find the area of major segment ALBQA. $(use\ \pi = 3.14)$



Ans:-



Given: Radius of the circle, r = 10 cm

Area of the circle

$= \pi(10)^2 = 3.14 \times 100 = 314\, cm^2$ ...(1)

Area of the sector OAPB

$= \frac{90°}{360°} \times \pi(10)^2$

$= \frac{1}{4} \times 314 \quad [From(1)]$

$= 78.5\, cm^2$

$Area\ of\ \Delta BOA$

$= \frac{1}{2} \times BO \times OA$

$= \frac{1}{2} \times 10 \times 10$

$= 50\, cm^2$

Area of the minor segment AQBP =

Area of the sector OAPB − Area of the triangle BOA

$= (78.5 - 50)\ cm^2$

$= 28.5\ cm^2$

Area of the major segment ALBQ

= Area of the circle − Area of the minor segment AQBP
= (314 − 28.5) cm^2
= 285.5 cm^2

Q15 Divide 56 in four parts in AP such that the ratio of the product of their extremes (1st and 4th) to the product of means (2nd and 3rd) is 5 : 6.

Ans:- Let the four terms of the AP be $a - 3d$, $a - d$, $a + d$ and $a + 3d$.
Given:
$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 56$
$\Rightarrow 4a = 56$
$\Rightarrow a = 14$
Also,

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6}$$

$$\Rightarrow \frac{a^2 - 9d^2}{a^2 - d^2} = \frac{5}{6}$$

$$\Rightarrow \frac{(14)^2 - 9d^2}{(14)^2 - d^2} = \frac{5}{6}$$

$$\Rightarrow \frac{196 - 9d^2}{196 - d^2} = \frac{5}{6}$$

$$\Rightarrow 1176 - 54d^2 = 980 - 5d^2$$

$$\Rightarrow 196 = 49d^2$$

$$\Rightarrow d^2 = 4$$

$$\Rightarrow d = \pm 2$$

When $d = 2$, the terms of the AP are 8, 12, 16, 20. When $d = -2$, the terms of the AP are 20, 18, 12, 8.

Q16 Solve the given quadratic equation for $x:\ 9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0.$

Ans:- = $9x^2 - 9(a+b)$

$x+\left(2a^2+5ab+2b^2\right)=0$

$\Rightarrow 9x^2-3\left\{\left(2a+b\right)+\left(a+2b\right)\right\}$

$+\left(2a^2 2+4ab+ab+2b2\right)=0$

$\Rightarrow 9x2-3\left\{\left(2a+b\right)+\left(a+2b\right)\right\}$

$x+\left\{2a\left(a+2b\right)+b\left(a+2b\right)\right\}=0$

$\Rightarrow 9x^2-3\left\{\left(2a+b\right)+\left(a+2b\right)\right\}$

$x+\left(2a+b\right)\left(a+2b\right)=0$

$\Rightarrow 9x2-3\left(2a+b\right)x-3\left(a+2b\right)$

$x+\left(2a+b\right)\left(a+2b\right)=0$

$\Rightarrow 3x\left\{3x-\left(2a+b\right)\right\}$

$-\left(a+2b\right)\left\{3x-\left(2a+b\right)\right\}=0$

$\Rightarrow \left\{3x-\left(a+2b\right)\right\}\left\{3x-\left(2a+b\right)\right\}=0$

$\Rightarrow 3x-\left(a+2b\right)=0$

$or\ 3x-\left(2a+b\right)=0$

$\Rightarrow 3x=a+2b\ or\ 3x=2a+b$

$\Rightarrow x=\frac{a+2b}{3}\ or\ x=\frac{2a+b}{3}$

Q17 A cylindrical tub, whose diameter is 12 cm and height 15 cm is full of ice-cream. The whole ice-cream is to be divided into 10 children in equal ice-cream cones, with conical base surmounted by hemispherical top. If the height of conical portion is twice the diameter of base, find the diameter of conical part of ice-cream cone.

Ans:- Let *R* and *H* be the radius and height of the cylindrical tub, respectively.

Given: Diameter of the cylindrical tub = 12 cm

∴ Radius, *R* = 6 cm

Height of the cylindrical tub, *H* = 15 cm

Volume of the ice-cream in the cylindrical tub

$=\pi R^2 H=\pi\times\left(6\right)^2\times 15=540\pi\, cm^3$

Let the diameter of the cone be *d* cm.

∴ Height of the conical portion = 2*d* cm

Radius of the hemispherical top = *d*2 cm

It is given that the ice-cream is divided among 10 children in equal ice-cream cones.
∴ Volume of the ice-cream in the cylindrical tub
= 10 × Volume of each ice-cream cone
⇒ Volume of the cylinder = 10 × (Volume of the cone + Volume of the hemisphere)

$$\Rightarrow 540\pi = 10\times\left[\frac{1}{3}\pi\times\left(\frac{d}{2}\right)^2\times 2d+\frac{2}{3}\pi\times\left(d^2\right)3\right]$$

$$\Rightarrow 540\pi = 10\times 13\pi\left(d^2\right)2\left(2d+2\times d2\right)$$

$$\Rightarrow 540\pi = \frac{5}{2}\pi d^3$$

$$\Rightarrow d^3 = \frac{540\times 2}{5} = 216$$

$$\Rightarrow d^3 = (6)^3$$

$$\Rightarrow d = 6$$

Therefore, the diameter of the conical part of the ice-cream cone is 6 cm.

Q18 A metal container, open from the top, is in the shape of a frustum of a cone of height 21 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of Rs 35 per litre. $\left[Use\ \pi = \frac{22}{7}\right]$**.**

Ans:- Given:
Height of the metal container, *h* = 21 cm
Radius of the lower end of the metal container, *r* = 8 cm
Radius of the upper end of the metal container, *R* = 20 cm

Volume of the metal container

$$= 13\pi h\left[R^2+Rr+r^2\right]$$

$$=\frac{1}{3}\times\frac{22}{7}\times 21\times\left[(20)^2+20\times 8+8^2\right]$$

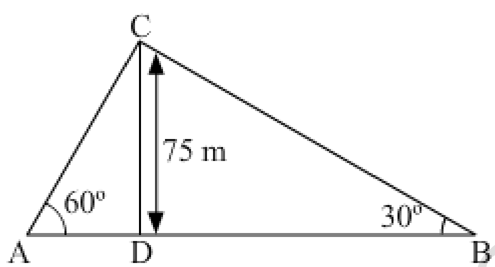
$$=\frac{1}{3}\times\frac{22}{7}\times 21\times 624$$

$$=13728\, cm^3$$

$$= \frac{13728}{1000} L \quad [\because 1L = 1{,}000 cm^3]$$
$$= 13.728 L$$

Now, Cost of 1 L of milk = Rs 35
∴ Cost of 13.728 L of milk = Rs 35 × 13.728 = Rs 480.48
Hence, the cost of the milk that can completely fill the container at the rate of Rs 35 per litre is Rs 480.48.

Q19 Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. Find the distance between the two men. $\left(Use\ \sqrt{3} = 1.73\right)$
Ans:-



Let CD be the height of the building.
So,
CD = 75 m
It is given that two men are at points A and B and they observe angles of elevation of the top of the building as 60° and 30°, respectively.

$$In\ \Delta ACD, tan 60° = \frac{CD}{AD}$$
$$\Rightarrow \sqrt{3} = \frac{75}{AD}$$
$$\Rightarrow AD = 25\sqrt{3} m$$
$$In\ \Delta BCD, tan 30° = \frac{CD}{BD}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BD}$$
$$\Rightarrow BD = 75\sqrt{3} m$$

∴ Distance between both the men = BD + AD =

$75\sqrt{3}\surd + 25\sqrt{3} = 100\sqrt{3}\,m$

$= 100 \times 1.73 = 173\,m$

Q20 A game consist of tossing a one-rupee coin 3 times and noting the outcome each time. Ramesh will win the game if all the tosses show the same result, (i.e. either all three heads or all three tails) and loses the game otherwise. Find the probability that Ramesh will lose the game.

Ans:- Elementary outcomes of tossing a one-rupee coin three times are as follows:

{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

∴ Total number of outcomes = 8

Ramesh will lose the game if the outcome is any of the following:

{HHT, HTH, THH, HTT, THT, TTH}

∴ Number of favourable outcomes = 6

Now,

Probability that Ramesh will lose the game

$= \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$

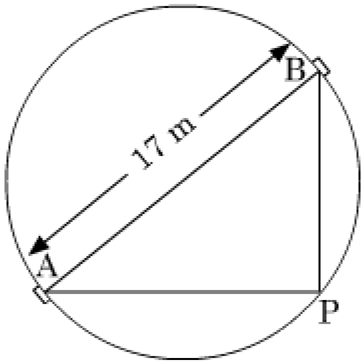
$= \frac{6}{8} = \frac{3}{4}$

Hence, the probability that Ramesh will lose the game is $\frac{3}{4}$

SECTION D

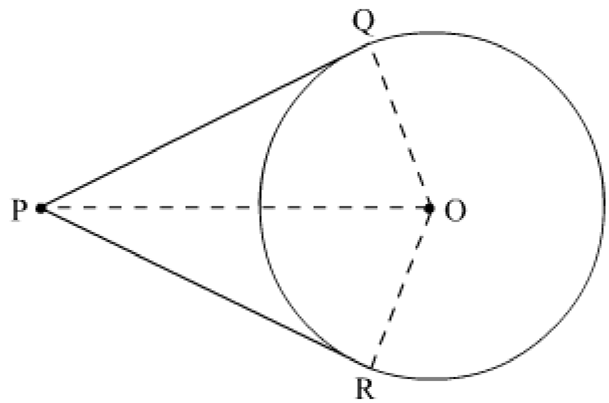
Q21 A pole has to be erected at a point on the boundary of a circular park of diameter 17 m in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Find the distances from the two gates where the pole is to be erected.

Ans:-



Let P be the required location of the pole.
Let the distance of the pole from the gate B be x m, i.e. BP = x m.
Now, the difference of the distances of the pole from the two gates = AP − BP (or, BP − AP) = 7 m. Therefore, AP = $(x + 7)$ m.
Since AB (diameter) = 17 m,
∠APB = 90° (Angle inscribed in a semicircle is a right angle.)
$AP^2 + PB^2 = AB^2$ (By Pythagoras' theorem)
Therefore,
$(x + 7)^2 + x^2 = 17^2$
$\Rightarrow x^2 + 14x + 49 + x^2 = 289$
$\Rightarrow 2x^2 + 14x - 240 = 0$
$\Rightarrow x^2 + 7x - 120 = 0$
$\Rightarrow x^2 + (15 - 8)x - 120 = 0$
$\Rightarrow x^2 + 15x - 8x - 120 = 0$
$\Rightarrow (x + 15)(x - 8) = 0$
$\Rightarrow x = -15$ or 8
Since x is the distance between the pole and the gate B, it must be positive.
So,
$x = 8$
Thus, the pole has to be erected on the boundary of the park at a distance of 8 m from the gate B and a distance of 15 m from the gate A.

Q22 Prove that the lengths of tangents drawn from an external point to a circle are equal.
Ans:- We have a circle with centre O. A point P lies outside the circle. Also, PQ and PR are two tangents to the circle.
To prove: PQ = PR
Construction: Join OP, OQ and OR.
Proof:



In ΔOQP and ΔORP,

$\angle OQP = \angle ORP = 90^\circ$ (Tangent is perpendicular to the radius through the point of contact.)
OQ = OR (Radii)
OP = OP (Common)
$\therefore \Delta OQP \cong \Delta ORP$ (By the RHS congruency criterion)
⇒ PQ = PR (Corresponding parts of congruent triangles)
Hence proved.

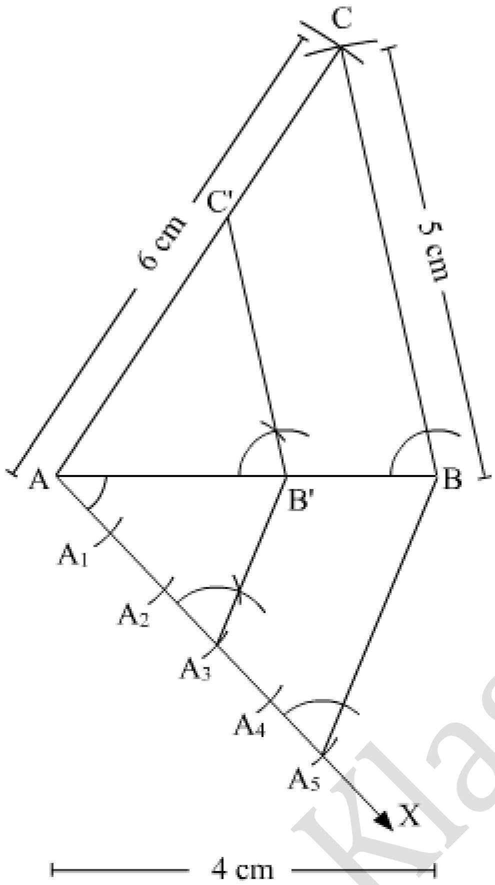
Q23 Draw a ΔABC in which AB = 4 cm, BC = 5 cm and AC = 6 cm. Then construct another triangle whose sides are 35 of the corresponding sides of ?ABC.
Ans:- Steps of Construction
Step 1
Draw a line segment AB = 4 cm. Taking point A as the centre and radius 6 cm, draw an arc. Similarly, taking point B as the centre and radius 5 cm, draw another arc. The arcs will intersect at point C. Now, join AC and BC to obtain the required triangle ABC.

Step 2
Draw a ray AX making an acute angle with line AB on the opposite side of vertex C.

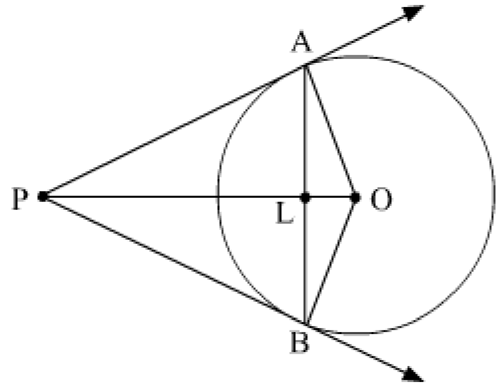
Step 3
Locate five points A_1, A_2, A_3, A_4 and A_5 on line AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$

Step 4
Join BA_5 and draw a line through A_3 parallel to BA_5 to intersect AB at point B'.

Step 5
Draw a line through B' parallel to line BC to intersect AC at C'.
AB'C' is the required triangle.



Q24 In fig. 6, AB is a chord of a circle, with centre O, such that AB = 16 cm and radius of circle is 10 cm. Tangents at A and B intersect each other at P. Find the length of PA.



Ans:- It is given that AB is the chord of the circle with centre O.
Now,
OP⊥ AB
Therefore, OP bisects AB.
AL = BL = 8 cm
Also,

$OL = \sqrt{(OA)^2 - (AL)^2} = \sqrt{10^2 - 8^2} = -\sqrt{36} = 6\,cm$

Now,
∠PAL + ∠OAL = 90° (Tangent is perpendicular to the radius through the point of contact.)
∠PAL + ∠APL = 90° (Sum of angles in ?APL is 180°.)
∴ ∠PAL + ∠OAL = ∠PAL + ∠APL
⇒∠OAL = ∠APL

$Now,\ in\ \Delta APL\ and\ \Delta OAL,$

∠PLA = ∠OLA (90° each)
∠APL = ∠OAL (Proved)

$\therefore \Delta APL \sim \Delta OAL (AA\ similarity)$

$\Rightarrow \frac{PA}{OA} = \frac{AL}{OL}$ (Corresponding sides are proportional.)

$\Rightarrow \frac{PA}{10} = \frac{8}{6}$

$\Rightarrow PA = \frac{40}{3} cm$

Thus, the length of the tangent $PA\ is\ \frac{40}{3}$ cm.

Q25 Find the positive value(s) of k for which quadratic equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ both will have real roots.

Ans:- The given quadratic equations are

$x^2 + kx + 64 = 0$ and

$x^2 - 8x + k = 0$

$x^2 + kx + 64 = 0$ has real roots.

$\therefore D = k^2 - 4 \times 1 \times 64 \geq 0$

$\Rightarrow k^2 - 16^2 \geq 0 \Rightarrow (k+16)(k-16) \geq 0$

Case 1

$k+16 \geq 0$ and $k-16 \geq 0$

$\Rightarrow k \geq -16$ and $k \geq 16$

$\Rightarrow k \geq 16 \quad .....(1)$

Case 2

$k+16 \leq 0$ and $k-16 \leq 0$

$\Rightarrow k \leq -16$ and $k \leq 16$

$\Rightarrow k \leq -16 \quad .....(2)$

From (1) and (2), we get

$k \in (-\infty, -16] \cup [16, \infty) \quad .....(3)$

Now, $x^2 - 8x + k = 0$ has real roots.

$\therefore D = (-8)^2 - 4 \times 1 \times k \geq 0$

$\Rightarrow 64 - 4k \geq 0 \Rightarrow -4k \geq -64$

$\Rightarrow k \leq \frac{-64}{-4}$

$\Rightarrow k \leq 16$

$\Rightarrow k \in (-\infty, 16] \quad .....(4)$

From (3) and (4), we get

$k \in (-\infty, -16] \cup \{16\}$

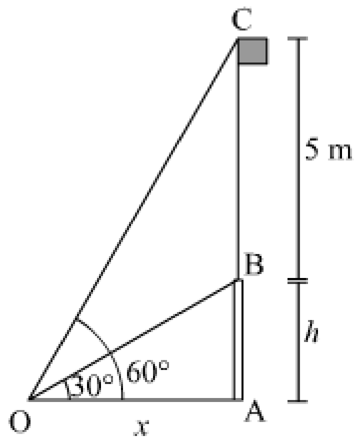
Since *k* is positive, it is 16.

Thus, the positive value of *k* for which both the equations will have real roots is 16.

Q26 A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 5 m. From a point on the ground the angles of elevation of the top and bottom of the flagstaff are 60° and 30° respectively. Find the

height of the tower and the distance of the point from the tower. $\left(take\ \sqrt{3} = 1.732\right)$

Ans:-



Let AB be the vertical tower and BC be the flagstaff.
Also, let O be the point on the ground from where the angles of elevation of the top and bottom of the flagstaff are 60° and 30°, respectively.
It is given that BC = 5 m.
Let the height of the vertical tower AB be *h* m and the distance of the point O from the tower be *x* m.

$In\ \Delta OAB,$

$$\tan 30° = \frac{AB}{OA} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow x = \sqrt{3}h \qquad .....(1)$$

$In\ \Delta OAC,$

$$\tan 60° = \frac{AC}{OA}$$

$$\Rightarrow \sqrt{3} = \frac{h+5}{x}$$

$$\Rightarrow x = \frac{h+5}{\sqrt{3}} \qquad .....(2)$$

From (1) and (2), we get

$$\sqrt{3}h = \frac{h+5}{\sqrt{3}}$$

$$\Rightarrow 3h = h+5$$

$$\Rightarrow 2h = 5$$

$$\Rightarrow h = \frac{5}{2} = 2.5$$

Thus, the height of the tower is 2.5 m.
Substituting *h* = 2.5 in (1), we get
$x=\sqrt{3}\times 2.5=1.732\times 2.5=4.33$
Therefore, the distance of the point O from the tower is 4.33 m.

Q27 Reshma wanted to save at least Rs 6,500 for sending her daughter to school next year (after 12 month.) She saved Rs 450 in the first month and raised her savings by Rs 20 every next month. How much will she be able to save in next 12 months? Will she be able to send her daughter to the school next year?
What value is reflected in this question.
Ans:- It is given that Reshma saved ?450 in the first month and raised her savings by ?20 every next month.
So, her savings are in an AP, with the first term (*a*) = ?450 and the common difference (*d*) = ?20.
We need to find her savings for 12 months, so *n* = 12.
We know that the sum of *n* terms of an AP is $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$.
Reshma's savings for 12 months:
$$S_{12}=\frac{12}{2}\left[2\times 450+(12-1)\times 20\right]$$
$$=6(900+220)$$
$$=6\times 1120$$
$$=6720$$

So, she will save ?6,720 in 12 months.
She needed to save at least ?6,500 for sending her daughter to school next year.
Since ?6,720 is greater than ?6,500, Reshma can send her daughter to school.
The question aims to encourage personal savings and emphasise the need of female education.

Q28 The co-ordinates of the points A, B and C are (6, 3), (−3, 5) and (4, −2) respectively. P(*x*, *y*) is any point in the plane. Show that $\frac{ar(PBC)}{ar(\Delta ABC)}=\left|\frac{x+y-2}{7}\right|$

Ans:- We know that the area of a triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_2(y_1-y_2)|$$

Now,

$$Area\ of\ \Delta\ PBC$$

$$=\frac{1}{2}\left|x[5-(-2)]+(-3)(-2-y)+4(y-5)\right|$$

$$=\frac{1}{2}|7x+7y-14|$$

$$=\frac{7}{2}|x+y-2| squareunits$$

$$Area\ of\ \Delta ABC$$

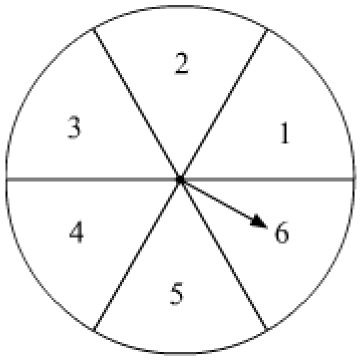
$$=\frac{1}{2}\left|6[5-(-2)]+(-3)(-2-3)+4(3-5)\right|$$

$$=\frac{1}{2}|42+15-8|=\frac{49}{2} squareunits$$

$$\therefore \frac{ar(\Delta PBC)}{ar(\Delta ABC)}=\frac{\frac{7}{2}|x+y-2|}{\frac{49}{2}}$$

$$\Rightarrow \frac{ar(\Delta PBC)}{ar(\Delta ABC)}=\frac{|x+y-2|}{7}=\left|\frac{x+y-2}{7}\right|$$

Q29 In fig. 7 is shown a disc on which a player spins an arrow twice. The fraction $\frac{a}{b}$ is formed, where 'a' is the number of sector on which arrow stops on the first spin and 'b' is the number of the sector in which the arrow stops on second spin. On each spin, each sector has equal chance of selection by the arrow. Find the probability that the fraction $\frac{a}{b}>1.$



Ans:- Elementary events associated with the experiment of spinning an arrow twice on a disc are

(1, 1), (1, 2), ...,(1, 6)
(2, 1), (2, 2), ...,(2, 6)
(3, 1), (3, 2), ...,(3, 6)
(4, 1), (4, 2), ...,(4, 6)
(5, 1), (5, 2), ...,(5, 6)
(6, 1), (6, 2), ...,(6, 6)

∴ Total number of elementary events = 36

It is given that *a* is the number of sector on which the arrow stops on the first spin and *b* is the number of the sector on which the arrow stops on the second spin.

Let *A* denote the event $"\frac{a}{b} > 1"$.

Now,

$\frac{a}{b} > 1 \Rightarrow a > b$

So, elementary events favourable to the event *A* are

(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4) and (6, 5)

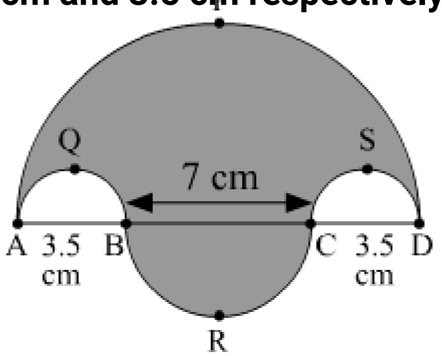
∴ Total number of favourable events = 15

Thus,

Required probability

$$= \frac{Total\ number\ of\ favourable\ events}{Total\ number\ of\ elementary\ events} = \frac{15}{36} = \frac{5}{12}$$

Q30 Find the area of the shaded region in Fig. 8, where APD, AQB, BRC and CSD are semi-circles of diameter 14 cm, 3.5 cm, 7 cm and 3.5 cm respectively. $\left[Use\ \pi = \frac{22}{7}\right]$



Ans:- Area of the shaded region
= Area of the semicircle with diameter AD + Area of the semicircle with diameter BC – Area of the semicircle with diameter AB – Area of the semicircle with diameter CD

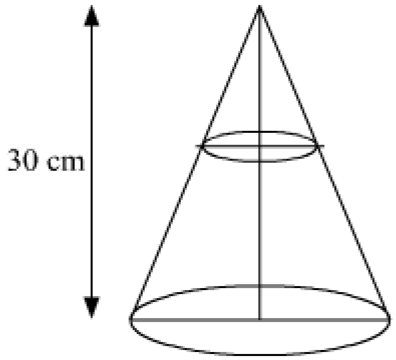
$$= \frac{1}{2}\pi\left(\frac{14}{2}\right)^2 + \frac{1}{2}\pi\left(\frac{7}{2}\right)^2 - \frac{1}{2}\pi\left(\frac{3.5}{2}\right)^2 - \frac{1}{2}\pi\left(\frac{3.5}{2}\right)^2$$

$$= \frac{1}{2}\times\frac{22}{7}\times\left(\frac{3.5}{2}\right)^2\times[16+4-1-1]$$

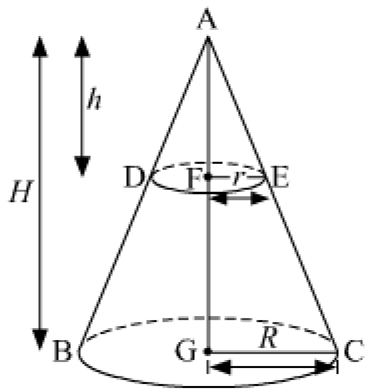
$$= \frac{1}{2}\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}\times 18 = 86.625\,cm^2$$

Q31 In fig. 9 is shown a right circular cone of height 30 cm. A small cone is cut off from the top by a plane parallel to the base. If the volume of the small cone is $\frac{1}{27}$ of the volume of cone, find at what height above the base is the

section made.



Ans:- Let the radius and height of the bigger cone be *R* and *H*, respectively.
Given: *H* = 30 cm
Let the radius and height of the smaller cone be *r* and *h*, respectively.



Now, in ?AFE and ?AGC,
∠AEF = ∠ACG (Corresponding angles)
∠AFE = ∠AGC (90° each)
$\therefore \Delta AFE \sim \Delta AGC \quad (AA\ similarity)$

$$\Rightarrow \frac{AF}{AG} = \frac{EG}{GC}$$

$$\Rightarrow \frac{h}{H} = \frac{r}{R} \qquad .....(1)$$

It is given that

$$Volume\ of\ the\ smaller\ cone\ =\ \frac{1}{27}\times\ Volume\ of\ the\ bigger\ cone$$

$$\Rightarrow 13\pi r^2 h = \frac{1}{27}\times\frac{1}{3}\pi R^2 H$$

$$\Rightarrow \left(\frac{r}{R}\right)^2 \times \frac{h}{H} = \frac{1}{27}$$

$$\Rightarrow \left(\frac{h}{H}\right)^2 \times \frac{h}{H} = \frac{1}{27} \qquad \left[Using\,(1)\right]$$

$$\Rightarrow \left(\frac{h}{H}\right)^3 = \frac{1}{27}$$

$$\Rightarrow \frac{h}{H} = \frac{1}{3}$$

$$\therefore h = \frac{1}{3}\times H = \frac{1}{3}\times 30 = 10\,cm$$

Now,
FG = AG − AF = 30 cm − 10 cm = 20 cm
Hence, the section is made 20 cm above the base.